

Distance/ Midpoint #9

NAME _____ DATE _____ PERIOD _____

11-5 Study Guide and Intervention (continued)
The Distance Formula

Find Coordinates If you know the coordinates of one point and only one coordinate of a second point, you can use the Distance Formula to find the missing coordinate of the second point.

Example 2 Find the value of a if the distance between the points at $(-3, -2)$ and $(a, -5)$ is 5 units.

- $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ Distance Formula
 $5 = \sqrt{(a - (-3))^2 + (-5 - (-2))^2}$ $(x_1, y_1) = (-3, -2)$, $(x_2, y_2) = (a, -5)$, and $d = 5$
 $5 = \sqrt{(a + 3)^2 + (-3)^2}$ Simplify.
 $5 = \sqrt{a^2 + 6a + 9 + 9}$ Evaluate squares.
 $5 = \sqrt{a^2 + 6a + 18}$ Simplify.
 $5^2 = (\sqrt{a^2 + 6a + 18})^2$ Square each side.
 $25 = a^2 + 6a + 18$ Simplify.
 $0 = a^2 + 6a - 7$ Subtract 25 from each side.
 $0 = (a + 7)(a - 1)$ Factor.
 $a + 7 = 0$ or $a - 1 = 0$ Zero Product Property
 $a = -7$ or $a = 1$

The value of a is -7 or 1 .

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11-5 Study Guide and Intervention
The Distance Formula

The Distance Formula The Pythagorean Theorem can be used to derive the Distance Formula shown below. The Distance Formula can then be used to find the distance between any two points in the coordinate plane.

Distance Formula The distance between any two points with coordinates (x_1, y_1) and (x_2, y_2) is given by $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

Example 1 Find the distance between the points at $(-5, 2)$ and $(4, 5)$.
Example 2 Jill draws a line segment from point $(1, 4)$ on her computer screen to point $(96, 49)$. How long is the segment?

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
 $= \sqrt{(96 - 1)^2 + (49 - 4)^2}$
 $= \sqrt{97^2 + 45^2}$
 $= \sqrt{9409 + 2025}$
 $= \sqrt{11,434}$

The segment is about 106.93 units long.

Find the distance between each pair of points whose coordinates are given. Express answers in simplest radical form and as decimal approximations rounded to the nearest hundredth if necessary.

- $(1, 5)$, $(3, 1)$ $2\sqrt{5}$; 4.47
- $(0, 0)$, $(6, 8)$ 10
- $(1, 5)$, $(-8, 4)$ $\sqrt{82}$; 9.06
- $(-1, 4)$, $(3, 2)$ $2\sqrt{5}$; 4.47
- $(0, 0)$, $(-3, 5)$ $\sqrt{34}$; 5.83
- $(3, 4)$, $(0, 0)$ 5
- $(-2, -5)$, $(0, 8)$ $\sqrt{173}$; 13.15
- $(-2, 0)$, $(-3, -9)$ $\sqrt{82}$; 9.06
- $(1, -1)$, $(3, -2)$ $\sqrt{5}$; 2.24
- $(-2, -7)$, $(-2, -2)$ $\sqrt{41}$; 6.40
- $(-2, -8)$, $(7, -3)$ $\sqrt{105}$; 10.30
- $(3, -4)$, $(-4, -4)$ 7
- $(2, -6)$, $(-7, 1)$ $\sqrt{130}$; 11.40
- $(3, -4)$, $(-4, -16)$ $\sqrt{193}$; 13.89
- $(-9, 0)$, $(-2, 5)$ $\sqrt{74}$; 8.60
- $(-3, -5)$, $(1, -8)$ 5

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 $5 = \sqrt{a^2 + 6a + 9 + 9}$ Evaluate squares.
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 $0 = a^2 + 6a - 7$ Subtract 25 from each side.
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 $a = -7$ or $a = 1$

The value of a is -7 or 1 .

Find the possible values of a if the points with the given coordinates are the indicated distance apart.

- $(1, a)$, $(3, -2)$; $d = \sqrt{5}$ 2 , $(0, 0)$, $(a, 4)$; $d = 5$ 3 , $(2, -1)$, $(a, 3)$; $d = 5$
 -1 or -3 3 or -3 -1 or 5
- $(1, -3)$, $(a, 2)$; $d = 25$ 5 , $(1, a)$, $(-2, 4)$; $d = 3$ 6 , $(3, -4)$, $(-4, a)$; $d = \sqrt{65}$
 -6 or 8 4 -8 or 0
- $(a, -4)$, $(-3, -2)$; $d = \sqrt{13}$ 8 , $(0, 3)$, $(3, a)$; $d = 3\sqrt{2}$ 9 , $(a, 3)$, $(2, -4)$; $d = \sqrt{74}$
 -6 or 0 0 or 6 -3 or 7
- $(-2, -5)$, $(a, -2)$; $d = 5$ 11 , $(3, 3)$, $(-1, a)$; $d = 5$ 12 , $(-1, -1)$, $(4, a)$; $d = \sqrt{41}$
 -6 or 2 0 or 6 -5 or 3
- $(a, 5)$, $(-1, 2)$; $d = \sqrt{45}$ 14 , $(4, -2)$, $(a, 0)$; $d = 2\sqrt{2}$ 15 , $(-2, 1)$, $(a, -3)$; $d = 2\sqrt{5}$
 -7 or 5 2 or 6 -4 or 0

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First 3 children...

1) $4\sqrt{7} + 2\sqrt{7} = 6\sqrt{7}$ (E)

13) $8\sqrt{11} + 6\sqrt{11} - \sqrt{11} = 13\sqrt{11}$ (D)

2) $8\sqrt{3} - 3\sqrt{3} = 5\sqrt{3}$ (H)

14) $2\sqrt{15} - 5\sqrt{3} + 7\sqrt{15} = 9\sqrt{15} - 5\sqrt{3}$ (T)

3) $2\sqrt{10} - 9\sqrt{10} = -7\sqrt{10}$ (Y)

15) $\sqrt{2} - 18\sqrt{7} - 5\sqrt{2} + 3\sqrt{7} = -4\sqrt{2} - 15\sqrt{7}$ (E)

4) $10\sqrt{5} + \sqrt{5} = 11\sqrt{5}$ (T)

16) $-9\sqrt{c} + 4\sqrt{d} - 4\sqrt{c} + 2\sqrt{d} = -13\sqrt{c} + 6\sqrt{d}$ (A)

5) $-3\sqrt{10} - 8\sqrt{10} = -11\sqrt{10}$ (O)

6) $7\sqrt{x} - 15\sqrt{x} = -8\sqrt{x}$ (N)

7) $\sqrt{8} + \sqrt{50} = 2\sqrt{2} + 5\sqrt{2} = 7\sqrt{2}$ (D)

17) $\sqrt{27} + \sqrt{75} + \sqrt{12} = 3\sqrt{3} + 5\sqrt{3} + 2\sqrt{3} = 10\sqrt{3}$ (H)

8) $-\sqrt{12} + \sqrt{300} = -2\sqrt{3} + 10\sqrt{3} = 8\sqrt{3}$ (T)

18) $\sqrt{63} - \sqrt{28} + \sqrt{700} = 3\sqrt{7} - 2\sqrt{7} + 10\sqrt{7} = 11\sqrt{7}$ (V)

9) $3\sqrt{20} + 8\sqrt{45} = 6\sqrt{5} + 24\sqrt{5} = 30\sqrt{5}$ (A)

10) $\sqrt{150} - 7\sqrt{24} = 5\sqrt{6} - 14\sqrt{6} = -9\sqrt{6}$ (T)

19) $5\sqrt{8} + 9\sqrt{200} + \sqrt{32} = 10\sqrt{2} + 90\sqrt{2} + 4\sqrt{2} = 104\sqrt{2}$ (A)

11) $-2\sqrt{40} - 5\sqrt{40} = -6\sqrt{10} - 10\sqrt{10} = -16\sqrt{10}$ (N)

20) $-2\sqrt{54} + 7\sqrt{150} + 3\sqrt{144} = -6\sqrt{6} + 35\sqrt{6} + 36 = 29\sqrt{6} + 36$ (V)

12) $3\sqrt{98} - 6\sqrt{18} = 21\sqrt{2} - 18\sqrt{2} = 3\sqrt{2}$ (W)

$$\textcircled{21} -4\sqrt{500} + 12\sqrt{44} - \sqrt{80}$$

$$\begin{array}{ccc} \swarrow \searrow & \swarrow \searrow & \swarrow \searrow \\ \sqrt{100} \sqrt{5} & \sqrt{4} \sqrt{11} & \sqrt{10} \sqrt{5} \end{array}$$

$$\boxed{-40\sqrt{5} + 12\sqrt{11} - 4\sqrt{5}} =$$

$$\boxed{-44\sqrt{5} + 12\sqrt{11} \text{ (N)}}$$

$$\textcircled{22} 8\sqrt{12t} + \sqrt{300t} - 2\sqrt{27t}$$

$$\begin{array}{ccc} \swarrow \searrow & \swarrow \searrow & \swarrow \searrow \\ \sqrt{4} \sqrt{3t} & \sqrt{100} \sqrt{3t} & \sqrt{9} \sqrt{3t} \end{array}$$

$$16\sqrt{3t} + 10\sqrt{3t} - 6\sqrt{3t} =$$

$$\boxed{20\sqrt{3t} \text{ (0)}}$$

$$\textcircled{23} -10\sqrt{9t} + 3\sqrt{36t} - \sqrt{50t}$$

$$\begin{array}{ccc} \swarrow \searrow & \swarrow \searrow & \swarrow \searrow \\ \sqrt{9} \sqrt{t} & \sqrt{36} \sqrt{t} & \sqrt{25} \sqrt{2t} \end{array}$$

$$\boxed{-30\sqrt{t} + 18\sqrt{t} - 5\sqrt{2t}} =$$

$$\boxed{-12\sqrt{t} - 5\sqrt{2t} \text{ (E)}}$$

$$\textcircled{24} 5\sqrt{160t} + 12\sqrt{75t} - 4\sqrt{250t}$$

$$\begin{array}{ccc} \swarrow \searrow & \swarrow \searrow & \swarrow \searrow \\ \sqrt{16} \sqrt{10t} & \sqrt{25} \sqrt{3t} & \sqrt{25} \sqrt{10t} \end{array}$$

$$\boxed{20\sqrt{10t} + 60\sqrt{3t} - 20\sqrt{10t}}$$

$$\boxed{60\sqrt{3t} \text{ (M)}}$$

ELEVATOR ... #1

$$\textcircled{1} \sqrt{2}(\sqrt{8} - 5) = \sqrt{16} - 5\sqrt{2} =$$

$$\boxed{4 - 5\sqrt{2} \text{ (m.v.)}}$$

$$\textcircled{2} \sqrt{3}(1 + \sqrt{27}) = \sqrt{3} + \sqrt{81} =$$

$$\boxed{\sqrt{3} + 9 \text{ (a.T)}}$$

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1) $(5-\sqrt{2})(5+\sqrt{2}) = 25-2 = 23$ (O)

2) $(7+\sqrt{3})(7-\sqrt{3}) = 49-3 = 46$ (E)

3) $(\sqrt{10-6})(\sqrt{10+6}) = 10-36 = -26$ (T)

4) $(\sqrt{10-\sqrt{6}})(\sqrt{10+\sqrt{6}}) = 10-6 = 4$ (K)

5) $(\sqrt{2+8})(\sqrt{2+3}) = 2+8\sqrt{2}+3\sqrt{2}+24$
 $= 26+11\sqrt{2}$ (A)

6) $(\sqrt{13+1})(\sqrt{13-5}) = 13+\sqrt{13}-5\sqrt{13}-5$
 $= 8-4\sqrt{13}$ (S)

7) $(10-\sqrt{15})(3-\sqrt{15}) = 18-3\sqrt{15}-6\sqrt{15}+15$
 $= 33-9\sqrt{15}$ (M)

8) $(9+\sqrt{7})^2 = 81+18\sqrt{7}+7 = 88+18\sqrt{7}$ (U)

9) $3\sqrt{2}(\sqrt{10}+\sqrt{2}) = 3\sqrt{20}+3\sqrt{4} = 6\sqrt{5}+6$ (D)

10) $2\sqrt{5}(7\sqrt{3}-\sqrt{10}) = 14\sqrt{15}-2\sqrt{50} = 14\sqrt{15}-10\sqrt{2}$ (L)

11) $5\sqrt{3}(2\sqrt{15}+\sqrt{8}) = 10\sqrt{45}+5\sqrt{24} = 30\sqrt{5}+10\sqrt{6}$ (C)

12) $3\sqrt{10}(4\sqrt{3}-2\sqrt{15}) = 12\sqrt{30}-6\sqrt{150} = 6\sqrt{6}-9\sqrt{10}$ (F)

13) $(8+3\sqrt{5})(1+2\sqrt{5}) = 8+3\sqrt{5}+10\sqrt{5}+6\sqrt{25}$
 $= 8+13\sqrt{5}+30$
 $= 38+13\sqrt{5}$

14) $(2\sqrt{7}+4)(5\sqrt{7}-11) = 10\sqrt{49}-22\sqrt{7}+20\sqrt{7}-44$
 $= 70-2\sqrt{7}-44$
 $= 26-2\sqrt{7}$ (H)

15 →

$$\begin{aligned} (5) \quad & (3\sqrt{10} - 5\sqrt{2})(2\sqrt{10} + 6\sqrt{2}) = \\ & = 6\sqrt{100} + 18\sqrt{20} - 10\sqrt{20} - 30\sqrt{4} \\ & = \cancel{60} + 8\sqrt{20} - \cancel{60} \\ & = 8\sqrt{20} \\ & = \boxed{16\sqrt{5} \text{ (I)}} \end{aligned}$$

Box 9 (Vampire)

⑥ $3\sqrt{2} + 8\sqrt{\frac{1}{2}}$
 $3\sqrt{2} + \frac{8\sqrt{1} \cdot \sqrt{2}}{\sqrt{2} \cdot \sqrt{2}}$
 $3\sqrt{2} + \frac{8\sqrt{2}}{2}$
 $3\sqrt{2} + 4\sqrt{2}$
 $7\sqrt{2}$

⑦ $10\sqrt{\frac{1}{5}} + \sqrt{45}$
 $\frac{10\sqrt{1} \cdot \sqrt{5}}{\sqrt{5} \cdot \sqrt{5}} + \sqrt{45}$
 $\frac{10\sqrt{5}}{5} \quad \sqrt{9} \cdot \sqrt{5}$
 $2\sqrt{5} + 3\sqrt{5}$
 $5\sqrt{5}$

⑧ $\left(\frac{7}{1}\right)^2 + \frac{\sqrt{7}}{2}$
 $\frac{2\sqrt{7}}{2} + \frac{1\sqrt{7}}{2} = \frac{3\sqrt{7}}{2}$

⑨ $\sqrt{3} - \sqrt{\frac{1}{3}}$
 $\sqrt{3} - \frac{\sqrt{1} \cdot \sqrt{3}}{\sqrt{3} \cdot \sqrt{3}}$
 $3\left(\frac{\sqrt{3}}{3}\right) - \frac{\sqrt{3}}{3}$
 $\frac{3\sqrt{3}}{3} - \frac{1\sqrt{3}}{3} = \frac{2\sqrt{3}}{3}$

⑩ $\sqrt{\frac{2}{3}} + 4\sqrt{\frac{1}{6}}$
 $\frac{\sqrt{2} \cdot \sqrt{3}}{\sqrt{3} \cdot \sqrt{3}} \quad \frac{4\sqrt{1} \cdot \sqrt{6}}{\sqrt{6} \cdot \sqrt{6}}$
 $2\left(\frac{\sqrt{6}}{3}\right) + \frac{4\sqrt{6}}{6}$
 $\frac{2\sqrt{6}}{3} + \frac{4\sqrt{6}}{6}$
 $\frac{4\sqrt{6}}{6} = \sqrt{6}$

⑪ $\sqrt{\frac{3}{4}} \cdot \sqrt{\frac{2}{3}} = \frac{\sqrt{6}}{\sqrt{12}} = \sqrt{\frac{1}{2}} = \frac{\sqrt{2}}{2}$
 $\frac{\sqrt{1} \cdot \sqrt{2}}{\sqrt{2} \cdot \sqrt{2}} = \frac{\sqrt{2}}{2}$

⑫ $\sqrt{\frac{7}{10}} \cdot \sqrt{\frac{2}{2}} = \frac{\sqrt{14}}{\sqrt{20}} \xrightarrow{\sqrt{4}}$
 $\frac{7 \cdot \sqrt{5}}{2\sqrt{5} \cdot \sqrt{5}} = \frac{7\sqrt{5}}{2(5)} = \frac{7\sqrt{5}}{10}$

⑬ $7\sqrt{10} - 2\sqrt{90} + 4\sqrt{\frac{1}{10}}$
 $7\sqrt{10} - 2\sqrt{9} \cdot \sqrt{10} + \frac{4\sqrt{1} \cdot \sqrt{10}}{\sqrt{10} \cdot \sqrt{10}}$
 $7\sqrt{10} - 6\sqrt{10} + \frac{2\sqrt{10}}{10} = \frac{2\sqrt{10}}{5}$
 $5\left(\frac{\sqrt{10}}{5}\right) + \frac{2\sqrt{10}}{5}$
 $\frac{5\sqrt{10}}{5} + \frac{2\sqrt{10}}{5} = \frac{7\sqrt{10}}{5}$

⑭ $6\sqrt{\frac{1}{2}} - 4\sqrt{\frac{1}{8}}$
 $\frac{6\sqrt{1} \cdot \sqrt{2}}{\sqrt{2} \cdot \sqrt{2}} - \frac{4\sqrt{1} \cdot \sqrt{8}}{\sqrt{8} \cdot \sqrt{8}}$
 $\frac{6\sqrt{2}}{2} - \frac{4\sqrt{8}}{8} \xrightarrow{\sqrt{4}}$
 $3\sqrt{2} - \sqrt{2} = \frac{-8\sqrt{2}}{8} = -\sqrt{2}$
 $2\sqrt{2}$

$$(15) \quad 8\sqrt{\frac{1}{4}} + \sqrt{24}$$

$$\frac{8\sqrt{1} \cdot \sqrt{6}}{\sqrt{4} \cdot \sqrt{6}}$$

$$\frac{8\sqrt{6}}{4}$$

$$4$$

$$\frac{4\sqrt{6}}{3} + \left(\frac{2\sqrt{6}}{1}\right)3$$

$$\frac{4\sqrt{6}}{3} + \frac{6\sqrt{6}}{3} = \boxed{\frac{10\sqrt{6}}{3}}$$

$$\begin{aligned} (21) \quad & -4\sqrt{500} + 12\sqrt{44} - \sqrt{80} \\ & \begin{array}{ccc} \swarrow & \searrow & \swarrow \\ \sqrt{100} & \sqrt{5} & \sqrt{4} \sqrt{11} \\ \downarrow & & \downarrow \\ -40\sqrt{5} & + 12\sqrt{11} & - 4\sqrt{5} \end{array} = \\ & \boxed{-44\sqrt{5} + 12\sqrt{11}} \quad (\text{N}) \end{aligned}$$

$$\begin{aligned} (22) \quad & 8\sqrt{12t} + \sqrt{300t} - 2\sqrt{27t} \\ & \begin{array}{ccc} \swarrow & \searrow & \swarrow \\ \sqrt{4} & \sqrt{3t} & \sqrt{100} \\ \downarrow & & \downarrow \\ 16\sqrt{3t} & + 10\sqrt{3t} & - 6\sqrt{3t} \end{array} = \\ & \boxed{20\sqrt{3t}} \quad (\text{O}) \end{aligned}$$

$$\begin{aligned} (23) \quad & -10\sqrt{4t} + 3\sqrt{36t} - \sqrt{50t} \\ & \begin{array}{ccc} \swarrow & \searrow & \swarrow \\ \sqrt{4} & \sqrt{t} & \sqrt{36} \\ \downarrow & & \downarrow \\ -30\sqrt{t} & + 18\sqrt{t} & - 5\sqrt{2t} \end{array} = \\ & \boxed{-12\sqrt{t} - 5\sqrt{2t}} \quad (\text{E}) \end{aligned}$$

$$\begin{aligned} (24) \quad & 5\sqrt{160t} + 12\sqrt{75t} - 4\sqrt{250t} \\ & \begin{array}{ccc} \swarrow & \searrow & \swarrow \\ \sqrt{16} & \sqrt{10t} & \sqrt{25} \sqrt{3t} \\ \downarrow & & \downarrow \\ 20\sqrt{10t} & + 60\sqrt{3t} & - 20\sqrt{10t} \end{array} \\ & \boxed{40\sqrt{3t}} \quad (\text{M}) \end{aligned}$$

ELEVATOR...

#10

$$\begin{aligned} (1) \quad & \sqrt{2}(\sqrt{8} - 5) = \sqrt{16} - 5\sqrt{2} = \\ & \boxed{4 - 5\sqrt{2}} \quad (\text{m.v}) \end{aligned}$$

$$\begin{aligned} (2) \quad & \sqrt{3}(1 + \sqrt{27}) = \sqrt{3} + \sqrt{81} = \\ & \boxed{\sqrt{3} + 9} \quad (\text{a.T}) \end{aligned}$$

$$\begin{aligned} (3) \quad & \sqrt{5}(\sqrt{15} + \sqrt{2}) = \sqrt{75} + \sqrt{10} \\ & \begin{array}{cc} \swarrow & \searrow \\ \sqrt{25} & \sqrt{3} \end{array} \\ & \boxed{5\sqrt{3} + \sqrt{10}} \quad (\text{f.C}) \end{aligned}$$

$$\begin{aligned} (4) \quad & 2\sqrt{7}(\sqrt{7} - 4) = 2\sqrt{49} - 8\sqrt{7} = \\ & \boxed{14 - 8\sqrt{7}} \quad (\text{k.J}) \end{aligned}$$

$$\begin{aligned} (5) \quad & 8\sqrt{3}(2\sqrt{3} + \sqrt{8}) = 16\sqrt{9} + 8\sqrt{24} = \\ & \begin{array}{cc} \swarrow & \searrow \\ \sqrt{4} & \sqrt{6} \end{array} \\ & \boxed{48 + 16\sqrt{6}} \quad (\text{n.E}) \end{aligned}$$

$$\begin{aligned} (6) \quad & \sqrt{18}(\sqrt{2} - \sqrt{18}) = 5\sqrt{36} - \sqrt{324} = \\ & \boxed{30 - 18} = 12 \quad (\text{H.g}) \end{aligned}$$

$$\begin{aligned} (7) \quad & (5\sqrt{2} + \sqrt{3})(\sqrt{2} + 2\sqrt{3}) = \\ & 5(2) + \sqrt{6} + 10\sqrt{6} + 2(3) = \\ & 10 + 11\sqrt{6} + 6 = \\ & \boxed{16 + 11\sqrt{6}} \quad (\text{N.B}) \end{aligned}$$

$$\begin{aligned} (8) \quad & (\sqrt{5} + 9\sqrt{2})(4\sqrt{5} - \sqrt{2}) = \\ & 4(5) + 36\sqrt{10} - \sqrt{10} - 9(2) = \\ & 20 + 35\sqrt{10} - 18 = \\ & \boxed{2 + 35\sqrt{10}} \quad (\text{k.P}) \end{aligned}$$

$$\begin{aligned} (9) \quad & (\sqrt{3} + 8)^2 = 3 + 16\sqrt{3} + 64 = \\ & \boxed{67 + 16\sqrt{3}} \quad (\text{b.H}) \end{aligned}$$

$$\begin{aligned} (10) \quad & (\sqrt{6} - \sqrt{3})^2 = 6 - 2\sqrt{18} + 3 = \\ & 9 - 2\sqrt{18} = 9 - 6\sqrt{2} \quad (\text{c.O}) \\ & \begin{array}{cc} \swarrow & \searrow \\ \sqrt{9} & \sqrt{2} \end{array} \end{aligned}$$

$$(7+\sqrt{2})(7-\sqrt{2}) = 49 - 2 = 47 \quad (18)$$

$$\underline{47 \text{ (I.F.)}}$$

$$\frac{-20}{\sqrt{16}-\sqrt{2}} \cdot \frac{\sqrt{16}+\sqrt{2}}{\sqrt{16}+\sqrt{2}} =$$

$$(12) (\sqrt{15}+\sqrt{6})(\sqrt{15}-\sqrt{6}) = 15 - 6 = 9$$

$$\underline{9 \text{ (n.T.)}}$$

$$\frac{-20\sqrt{16} - 20\sqrt{2}}{16 - 2} = \frac{-20\sqrt{16} - 20\sqrt{2}}{14} =$$

$$\underline{-5\sqrt{16} - 5\sqrt{2} \text{ (9.L.)}}$$

$$(13) \frac{4}{\sqrt{5}+\sqrt{2}} \cdot \frac{\sqrt{5}-\sqrt{2}}{\sqrt{5}-\sqrt{2}} = \frac{4\sqrt{5}-4\sqrt{2}}{5-2} =$$

$$\underline{\frac{4\sqrt{5}-4\sqrt{2}}{3} \text{ (C.T.)}}$$

$$(19) \frac{10}{\sqrt{22}+4} \cdot \frac{\sqrt{22}-4}{\sqrt{22}-4} =$$

$$\frac{10\sqrt{22}-40}{22-16} = \frac{10\sqrt{22}-40}{6}$$

$$(14) \frac{15}{\sqrt{11}-\sqrt{10}} \cdot \frac{\sqrt{11}+\sqrt{10}}{\sqrt{11}+\sqrt{10}} = \frac{15\sqrt{11}+15\sqrt{10}}{11-10} =$$

$$\underline{\frac{5\sqrt{22}-20}{3} \text{ (n.S.)}}$$

$$\frac{15\sqrt{11}+15\sqrt{10}}{5} = \frac{5(3\sqrt{11}+3\sqrt{10})}{5} =$$

$$(20) \frac{\sqrt{5}}{3-\sqrt{8}} \cdot \frac{3+\sqrt{8}}{3+\sqrt{8}} =$$

$$\underline{3\sqrt{11}+3\sqrt{10} \text{ (I.R.)}}$$

$$\frac{3\sqrt{5} + \sqrt{40}}{9-8} = \frac{3\sqrt{5} + \sqrt{40}}{1}$$

$$\downarrow \quad \downarrow$$

$$\sqrt{4} \quad \sqrt{10}$$

$$(15) \frac{9}{\sqrt{3}+1} \cdot \frac{\sqrt{3}-1}{\sqrt{3}-1} = \frac{9\sqrt{3}-9}{3-1} =$$

$$\underline{3\sqrt{5} + 2\sqrt{10} \text{ (j.N.)}}$$

$$\underline{\frac{9\sqrt{3}-9}{2} \text{ (n.D.)}}$$

$$(16) \frac{36}{4-\sqrt{7}} \cdot \frac{4+\sqrt{7}}{4+\sqrt{7}} = \frac{144+36\sqrt{7}}{16-7} =$$

$$\frac{144+36\sqrt{7}}{9} = \underline{16 + 4\sqrt{7} \text{ (n.L.)}}$$

$$(17) \frac{56}{\sqrt{3}+\sqrt{10}} \cdot \frac{\sqrt{3}-\sqrt{10}}{\sqrt{3}-\sqrt{10}} = \frac{56\sqrt{3}-56\sqrt{10}}{3-10} =$$

$$\frac{56\sqrt{3}-56\sqrt{10}}{-7} = \underline{-8\sqrt{3}+8\sqrt{10} \text{ (I.H.)}}$$