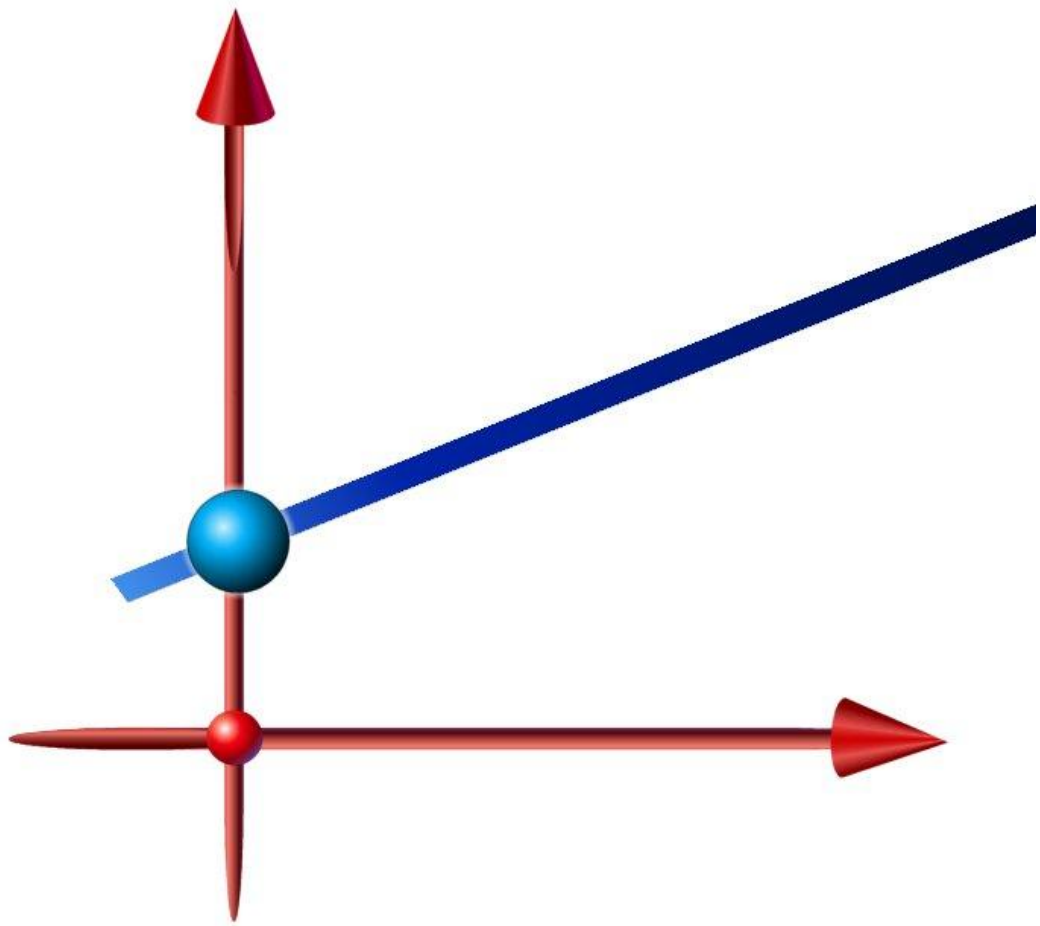
8.EE

EduTron Corporation

Draft for NYSED NTI Use Only



CHALLENGING PROBLEMS AND TASKS

8.EE.6 DERIVING EQUATIONS FOR LINES WITH NON-ZERO Y-INTERCEPTS

Development from y = mx to y = mx + b

DRAFT 2012.11.29

Table of Contents

I. Challenging Problems from EduTron

Use with Care!

Before you dive into the first 12 problems, please be fully aware that these exemplars are provided as templates and models for you to **develop your own capacity to produce more** problems that are of the same caliber.

Dissect these problems carefully with your peers to build up your knowledge and skills so you can create similar "problems *with teeth*." The process of developing problems with rigor and depth is serious fun. Most of all, it will further improve your craft and refine your taste.

It is important to know your students well so you can create, find, or modify problems that best suit their needs. **Do not just rush to unleash these problems on your students! First, make sure YOU are really taking full advantage of them!**

Thank you.

Read me upside down, too.

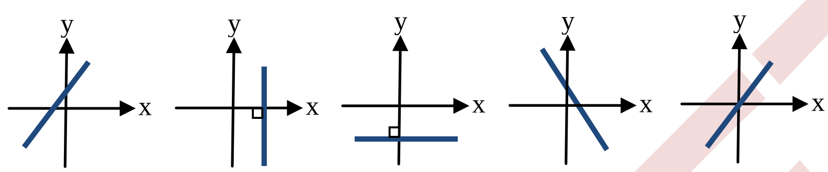
The EduTron Team

I. Challenging Problems from EduTron

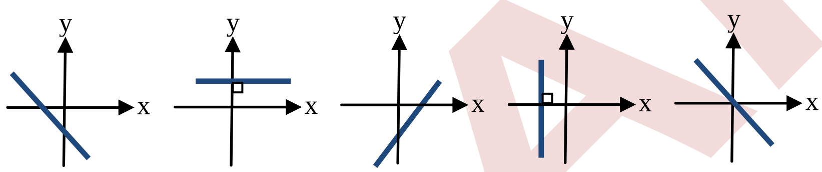
These problems could be used to make the Common Core Rigor and Mathematical Practice Standards come alive!

Fluency Concept Application MP1 MP2 MP3 MP4 MP5 MP6 MP7 MP8

1. Given $ab > 0$, a and b are real numbers, which graph can be described by $x + ay = b$? Why?



A B C D E



F G H I J

2. A line passes through (2, 4) and (-2, 2). Find the value of *y* if (6, *y*) lies on the line.

3. The vertices of a right triangle are P(-3, 4), Q(-3, -4) and R(7, -4). Explain why the point S(2, 0) lies on (the sides of) the triangle.

4. Find the equation of the image when the line ($y = m\,x + k$) is reflected about (a) the *x*-axis; and (b) the *y*-axis.

5. Graph $\frac{x}{3}+\frac{y}{5}=1$ (2) What is the slope of the line? (3) Discuss the significance of *a* and *b* for the graph of $\frac{x}{a}+\frac{y}{b}=1$.

6. For a line with a positive slope, which of the following are correct?
   a. The x-intercept and y-intercept must have opposite signs.
   b. The x-intercept and y-intercept can both be 0.
   c. The x-intercept and y-intercept can have opposite signs.
   d. The slope is equal to y-intercept/x-intercept.
   e. The product of x-intercept and y-intercept can be positive.
   f. None of the above.

7. What is the area of the triangle formed by the x-axis, the y-axis and the line $ax+by+c=0$, where a, b, and c are not 0.

8. A line with slope of –3 passes through (-8, *p*) and (2, 3*p*). Find the value of *p*.

9. Graph a family of lines of the form $y = 3x + c$ on the same *x*-*y* plane, where *c* is any real number. Describe the pattern in the graph.

10. Graph a family of lines of the form $y = cx - 5$ on the same *x*-*y* plane, where *c* is any real number. Describe the pattern in the graph.

11. Graph a family of lines of the form $y = cx + c$ on the same *x*-*y* plane, where *c* is any real number. Describe the pattern in the graph.

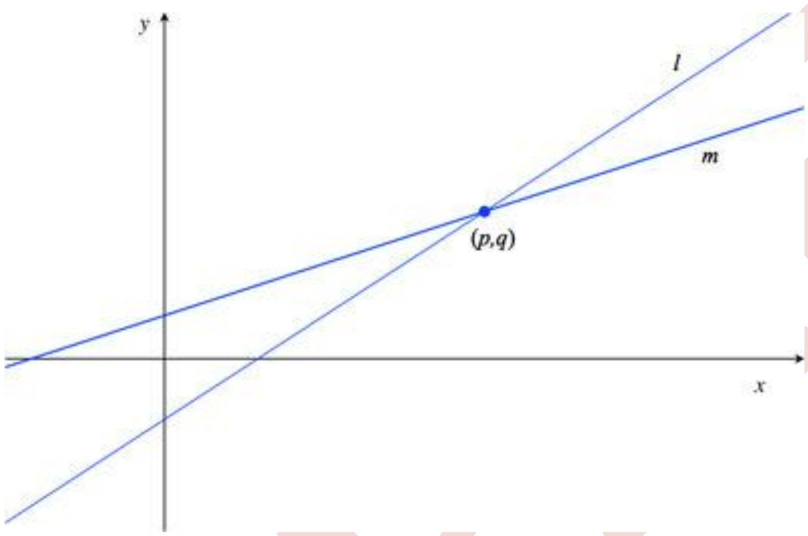
12. Consider the line $\frac{x}{a} + \frac{y}{b} = 1$, where $a \neq 0$ and $b \neq 0$. If the line does not pass the 4th quadrant, in what quadrant is the point ($a - b$, ab) located?

II. Classroom Tasks from Illustrative Mathematics

1. 8.EE Equations of Lines

Alignment 1: 8.EE.B

The figure below shows the lines l and m described by the equations $4x-y=c$ and $y=2x+d$ respectively, for some constants c and d. They intersect at the point (p,q).



1. **How can you interpret c and d in terms of the graphs of the equations above?**
2. **Imagine you place the tip of your pencil at point (p,q) and trace line l out to the point with x-coordinate $p+2$. Imagine I do the same on line m. How much greater would the y-coordinate of your ending point be than mine?**

Commentary:

This task requires students to use the fact that on the graph of the linear equation $y=ax+c$, the y-coordinate increases by c when x increases by one. Specific values for c and d were left out intentionally to encourage students to use the above fact as opposed to computing the point of intersection, (p,q), and then computing respective function values to answer the question.

Solution:

1. If we put the equation $4x-y=c$ in the form $y=4x-c$, we see that the graph has slope 4. The slope of the graph of $y=2x+d$ is 2. So the steeper line, l, is the one with equation $y=4x-c$, and therefore $-c$ is the y-coordinate of the point where l intersects the y-axis. The other line, m, is the one with equation $y=2x+d$, so d is the y-coordinate of the point where m intersects the y-axis.
2. The line l has slope 4. So on l, each increase of one unit in the x-value produces an increase of 4 units in the y-value. Thus an increase of 2 units in the x-value produce an increase of $2 \cdot 4=8$ units in the y-value. The line m has slope 2. So on $L2$, each increase of 1 unit in the x-value produces an increase of 2 units in the y-value. Thus an increase of 2 units in the x-value produces an increase of $2 \cdot 2=4$ units in the y-value.

   Thus your y-value would be $8–4=4$ units larger than my y-value.

2. 8.EE Find the Change

Alignment 1: 8.EE.B

---

1. **The table below shows two coordinate pairs (*x*,*y*) that satisfy the equation *y* = *mx* + *b* for some numbers *m* and *b*.**

x	*y*
2	*y*1
5	*y*2

   1. **If *m* = 7, determine possible values for *y*1 and *y*2. Explain your choices.**
   2. **Find another pair of *y*-values that could work for *m* = 7. Explain why they would work. How do these *y*-values compare to the first pair you found for *m* = 7?**
   3. **Use the same *x*-values in the table and find possible values for *y*1 and *y*2 if *m* = 3. Explain your choices.**
   4. **Find another pair of *y*-values that could work for *m* = 3. Explain why they would work. How do these *y*-values compare to the first pair you found for *m* = 3?**

2. **Each of the three tables below shows two coordinate pairs (*x*,*y*) that satisfy the equation *y* = *mx* + *b* for some numbers *m* and *b*. If *m* = 3 in each case, find possible values for *y*1 and *y*2 for each pair of *x*-values given.**

i

x	*y*
4	*y*1
9	*y*2

ii

x	*y*
2	*y*1
13	*y*2

iii

x	*y*
-1	*y*1
14	*y*2

iv. Suppose we take all six *x*-values from the three tables above. Can you find six corresponding *y*-values so that all the coordinate pairs satisfy the same equation if *m*=3? Fill out the table below and explain how you know they will all work with the same equation.

x	*y*
4	
9	
2	
13	
-1	
14	

Commentary:

This task is adapted from one I have used with my eighth grade students to help them solidify their understanding of linear functions and push them to be more fluent in their reasoning about slope and *y*-intercepts. This task has also produced a reasonable starting place for discussing point-slope form of a linear equation.

With some instructional effort, this task can produce some nice classroom discussion. If adapted slightly (see comments in the solution), the task could be used as an assessment if placed properly in the curriculum. This task can also be extended so that the unknown values are *x*-values rather then *y*-values. There is also opportunity to discuss vertical and horizontal shifts if desired since all values that satisfy the tables for a given slope will be on lines that are parallel to one another.

Although this task provides information about linear relationships represented by tables, a strong connection can be made to both the equations and the graphs. By pressing students to create multiple representations and then focusing on their graphs, similar triangles can be used to verify the slopes and the equations. Students become comfortable with tables and representing the differences along the sides of the tables; maintaining the connection to what those differences mean graphically can assist them in building their understanding of slope, lines, and linear equations.

This task was submitted by Travis Lemon for the first IMP task writing contest 2011/12/12-2011/12/18.

Solution:

a. This task allows for multiple approaches and solutions:

- Students often will choose 14 and 35 when $m=7$ and 6 and 15 when $m=3$. This comes from a direct relationship with an initial value (y-intercept) of zero.
- Others chose $y1$ to be zero in both tables and then determine the second y-value by multiplying the slope with the difference in the x-values. This becomes their second y-value.
- Yet more fluent students realize that the difference in x-values multiplied by the slope would equal the difference in the y-values. Therefore, the y-values on the first table could be any values that have a difference of 21 ($y2>y1$), while the y-values used to satisfy a slope of 3 could be any values with a difference of 9 ($y2>y1$).

Part (ii) asks students to compare the pairs of y-values they found in parts (i) and (ii); this question is open-ended enough that it doesn't make a good assessment question, but it does make a good opener for discussion. While students might come up with a number of different ways of comparing them, the goal is to highlight that regardless of the values they found, the difference will always be the same. If no students make this observation by the time they get to part (iv), the teacher can point it out. Ideally, the discussion of the different approaches can lead to a solidification of the following relationship:

$$(\text{change in x–values})m=(\text{change in y-values})$$

and then a more formal

$$(x2-x1)m=(y2-y1)$$

b. There are many different solutions for parts (b)(i)–(iii); in all cases the difference between the *x*-values is 3 times the difference in the *y*-values. The final question (part iv) presses students once again to further solidify their understanding of slope by asking them to pick all six *y*-values so that all the pairs will satisfy the same equation.

Instructional opportunities arise as students justify their choices. Helping students focus on the ratio of the change in *y* ("rise") to the change in *x* ("run") and encouraging them to draw triangles and explain why they are similar can facilitate students' understandings. Teachers should capitalize on any opportunity to use multiple representations; strategically posed questions allow for students to make more connections. For example,

- "Can you simply take the values used in the first three parts and use them to fill in the table in the last part? If so, why? If not, why not?"
- "What would change if we used a negative slope?"
- "If someone decided to adjust one of the *y*-values they chose by 5, would any of the other values need to change? Why or why not? How would they need to change?"

3. 8.EE DVD Profits, Variation 1

Alignment 1: 8.EE.B

---

a. **DVDs can be made in a factory in New Mexico at the rate of 20 DVDs per $3, but the factory costs $80,000 to build. If they make 1 million DVDs, what is the unit cost per DVD?**
b. **DVDs can be made in a factory in Colorado at the rate of 10 DVDs per $1.50, but the factory costs $100,000 to build. If they make 1 million DVDs, what is the unit cost per DVD?**
c. **How much can a buyer save on a million DVDs by buying DVDs from New Mexico instead of DVDs from Colorado?**
d. **Find an equation for the cost of making *x* number of DVDs in the factory in New Mexico.**
e. **Find an equation for the cost of making *x* number of DVDs in the factory in Colorado.**

Commentary:

The first two problems ask for the unit cost per DVD for making a million DVDs. Even though each additional DVD comes at a fixed price, the overall cost per DVD changes with the number of DVDs produced because of the startup cost of building the factory.

Solution: Working with unit rates

If DVDs didn't need a factory, the DVDs from New Mexico would just be 20 DVDs per $3. The cost per DVD would be $.15 per DVD, or 15 cents per DVD. However, we have to factor in the factory cost.

The DVDs from Colorado, without the factory, would be 10 DVDs per $1.50, so the unit cost would also be $.15 per DVD, or 15 cents per DVD, the same as in the factory in New Mexico! Consequently, the only difference in the cost of DVDs between New Mexico and Colorado is the cost of the factory, which costs $20,000 more in Colorado, no matter how many DVDs are involved.

a. The cost of 1 million DVDs from New Mexico is

$$80{,}000 \text{ dollars} + 1{,}000{,}000 \text{ DVDs} \cdot \frac{3\ dollars}{20\ DVDs}$$

$$= 80{,}000 \text{ dollars} + 50{,}000 \cdot 3 \text{ dollars}$$

$$= 80{,}000 \text{ dollars} + 150{,}000 \text{ dollars} = 230{,}000 \text{ dollars}$$

The unit cost per DVD is:

$$\frac{230{,}000\ dollars}{1{,}000{,}000\ DVDs} = .23\ \frac{dollars}{DVD} = .23\ \frac{dollars}{DVD} \cdot \frac{100\ cents}{dollar} = 23\ \frac{cents}{DVD}$$

So the unit cost is 23 cents per DVD.

b. The cost of 1 million DVDs from Colorado is

$$100{,}000 \text{ dollars} + 1{,}000{,}000 \text{ DVDs} \cdot \frac{1.5\ dollars}{10\ DVDs}$$

$$= 100{,}000 \text{ dollars} + 100{,}000 \cdot 1.5 \text{ dollars}$$

$$= 100{,}000 \text{ dollars} + 150{,}000 \text{ dollars} = 250{,}000 \text{ dollars}$$

The unit cost per DVD is:

$$\frac{250{,}000\ dollars}{1{,}000{,}000\ DVDs} = .25\ \frac{dollars}{DVD} = .25\ \frac{dollars}{DVD} \cdot \frac{100\ cents}{dollar} = 25\ \frac{cents}{DVD}$$

So the unit cost is 25 cents per DVD.

c. The difference is 2 cents per DVD. For a million DVDs, the savings would be:

$$1{,}000{,}000 \text{ DVDs} \cdot \frac{2\ cents}{DVD} \cdot \frac{dollar}{100\ cents} = 20{,}000 \text{ dollars.}$$

Of course, the rate of 10 DVDs per 1.5 dollars is the same as 20 DVDs per 3 dollars, so the only difference is the startup cost difference, $20,000, no matter how many DVDs are made, 0 on up! We have also already computed the cost of a million DVDs from each state, so all we really had to do was subtract the cost of the DVDs from Colorado from the cost of the DVDs from New Mexico,

$$250{,}000 \text{ dollars} - 230{,}000 \text{ dollars} = 20{,}000 \text{ dollars.}$$

d. The cost *C* of making *x* DVDs in New Mexico is

$$C = 80{,}000 \text{ dollars} + \frac{3\ dollars}{20\ DVDs} \cdot x \text{ DVDs}$$

$$C = (80{,}000 + \tfrac{3}{20}x) \text{ dollars}$$

$$C = (80{,}000 + .15x) \text{ dollars.}$$

e. The cost C for making x DVDs in Colorado is

$$C = 100{,}000 \text{ dollars} + \frac{1.5\ dollars}{10\ DVDs} \cdot x \text{ DVDs}$$

$$C = (100{,}000 + \frac{1.5}{10}x) \text{ dollars}$$

$$C = (100{,}000 + .15x) \text{ dollars.}$$

Solution: Working backwards

If we find the equations asked for in parts (d) and (e) first, we might have saved ourselves a little work for parts (a) and (b).

If we make x DVDs and we want the unit cost per DVD, we could just use the equation for the cost for x DVDs in New Mexico:

$$\frac{(80{,}000 + .15x)\ dollars}{x\ DVDs} = (\frac{80{,}000}{x} + .15)\ \frac{dollars}{DVD}\ .$$

Or, in terms of cents:

$$(\frac{80{,}000}{x} + .15)\ \frac{dollars}{DVD} \times \frac{100\ cents}{dollar} = (\frac{8{,}000{,}000}{x} + 15)\ \frac{cents}{DVD}\ .$$

Now, to do part (a), we just plug in 1,000,000 and we get that one DVD costs (8+15) cents = 23 cents.

If we make x DVDs and we want the unit cost per DVD in Colorado, we could use the equation for the cost for x DVDs in Colorado:

$$\frac{(100{,}000 + .15x) dollars}{x\ DVD} = (\frac{100{,}000}{x} + .15)\ \frac{dollars}{DVD}\ .$$

Or, in terms of cents:

$$(\frac{100{,}000}{x} + .15)\ \frac{dollars}{DVD} \times \frac{100\ cents}{dollar} = (\frac{10{,}000{,}000}{x} + 15)\ \frac{cents}{DVD}.$$

Now, to do part (b), we just plug in 1,000,000 and we get that one DVD costs (10 + 15) cents = 25 cents.

III. Selected 8.EE Problems for NYS

[To be completed in the future]

IV. PARCC 6.RP Sample Item

[To be completed in the future]