

Distance/ Midpoint #9

NAME _____ DATE _____ PERIOD _____

11-5 Study Guide and Intervention (continued)

The Distance Formula

Find Coordinates If you know the coordinates of one point and only one coordinate of a second point, you can use the Distance Formula to find the missing coordinate of the second point.

Example 2 Find the value of a if the distance between the points at $(-3, -2)$ and $(a, -5)$ is 5 units.

- $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ Distance Formula
 $5 = \sqrt{(a - (-3))^2 + (-5 - (-2))^2}$ $(x_1, y_1) = (-3, -2)$, $(x_2, y_2) = (a, -5)$, and $d = 5$
 Simplify.
 $5 = \sqrt{(a + 3)^2 + (-3)^2}$
 Evaluate squares.
 $5 = \sqrt{a^2 + 6a + 9 + 9}$
 Simplify.
 $5 = \sqrt{a^2 + 6a + 18}$
 Square each side.
 $5^2 = (a^2 + 6a + 18)$
 $25 = a^2 + 6a + 18$
 Subtract 25 from each side.
 $0 = a^2 + 6a - 7$
 Factor.
 $0 = (a + 7)(a - 1)$
 $a + 7 = 0$ or $a - 1 = 0$
 $a = -7$ or $a = 1$
 Zero Product Property

The value of a is -7 or 1 .

Example 3 Find the possible values of a if the points with the given coordinates are the indicated distance apart.

- $(1, a), (3, -2); d = \sqrt{5}$
 -1 or -3
 $2. (0, 0), (a, 4); d = 5$
 3 or -3
 $3. (2, -1), (a, 3); d = 5$
 -1 or 5
- $(1, -3), (a, 2); d = 25$
 -6 or 8
 $5. (1, a), (-2, 4); d = 3$
 4
 $6. (3, -4), (-4, a); d = \sqrt{65}$
 -8 or 6
- $(a, -4), (-3, -2); d = \sqrt{13}$
 0 or 6
 $8. (0, 3), (3, a); d = 3\sqrt{2}$
 $9. (a, 3), (2, -4); d = \sqrt{74}$
 -3 or 7
- $(-2, -5), (a, -2); d = 5$
 0 or 6
 $11. (3, 3), (-1, a); d = 5$
 $12. (-1, -1), (4, a); d = \sqrt{41}$
 -5 or 3
- $(a, 5), (-1, 2); d = \sqrt{45}$
 -7 or 5
 $14. (4, -2), (a, 0); d = 2\sqrt{2}$
 2 or 6
 $15. (-2, 1), (a, -3); d = 2\sqrt{5}$
 -4 or 0

© Glencoe/McGraw-Hill

Glencoe Algebra 1

668

NAME _____ DATE _____ PERIOD _____

11-5 Study Guide and Intervention

The Distance Formula

The Distance Formula The Pythagorean Theorem can be used to derive the Distance Formula shown below. The Distance Formula can then be used to find the distance between any two points in the coordinate plane.

The distance between any two points with coordinates (x_1, y_1) and (x_2, y_2) is given by
 $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

Example 1 Find the distance between the points at $(-5, 2)$ and $(4, 5)$.
 $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ Distance Formula
 $d = \sqrt{(-5 - 4)^2 + (5 - 2)^2}$ $(x_1, y_1) = (-5, 2)$, $(x_2, y_2) = (4, 5)$
 Simplify.
 $d = \sqrt{9^2 + 3^2}$
 Evaluate squares and simplify.
 $d = \sqrt{81 + 9}$
 $d = \sqrt{90}$

The distance is $\sqrt{90}$, or about 9.49 units.

Example 2 Jill draws a line segment from point $(1, 4)$ on her computer screen to point $(96, 49)$. How long is the segment?
 $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
 $d = \sqrt{(96 - 1)^2 + (49 - 4)^2}$
 $d = \sqrt{95^2 + 45^2}$
 $d = \sqrt{9409 + 2025}$
 $d = \sqrt{11,434}$
 The segment is about 106.93 units long.

Example 3 Find the distance between each pair of points whose coordinates are given. Express answers in simplest radical form and as decimal approximations rounded to the nearest hundredth if necessary.

- $(1, 5), (3, 1)$
 $2\sqrt{5}; 4.47$
- $(0, 0), (6, 8)$
 10
- $(1, 5), (-8, 4)$
 $\sqrt{82}; 9.06$
- $(-1, 4), (3, 2)$
 $2\sqrt{5}; 4.47$
- $(0, 6), (-3, 5)$
 $\sqrt{34}; 5.83$
- $(3, 4), (0, 0)$
 5
- $(-2, -5), (0, 8)$
 $\sqrt{173}; 13.15$
- $(1, -1), (3, -2)$
 $\sqrt{5}; 2.24$
- $(-2, -7), (-2, -2)$
 $\sqrt{41}; 6.40$
- $(-2, 0), (-3, -9)$
 $\sqrt{82}; 9.06$
- $(3, -4), (-4, -16)$
 $\sqrt{193}; 13.89$
- $(-9, 0), (-2, 5)$
 $\sqrt{74}; 8.60$
- $(-3, -5), (1, -8)$
 5
- $(1, -3), (-8, 21)$
 $\sqrt{657}; 25.63$
- $(-2, -8), (7, -3)$
 $\sqrt{105}; 10.30$
- $(3, -4), (-4, -4)$
 7
- $(2, -6), (-7, 1)$
 $\sqrt{130}; 11.40$
- $(3, -4), (-4, -16)$
 $\sqrt{193}; 13.89$
- $(-9, 0), (-2, 5)$
 $\sqrt{74}; 8.60$
- $(-3, -5), (1, -8)$
 5

© Glencoe/McGraw-Hill

Glencoe Algebra 1

667

Lesson 11-5

First 3 children...

1) $4\sqrt{7} + 2\sqrt{7} = 6\sqrt{7}$ (E)

13) $8\sqrt{11} + 4\sqrt{11} - \sqrt{11} = 13\sqrt{11}$ (D)

2) $8\sqrt{3} - 3\sqrt{3} = 5\sqrt{3}$ (H)

14) $2\sqrt{15} - 5\sqrt{3} + 7\sqrt{15} = 9\sqrt{15} - 5\sqrt{3}$ (T)

3) $2\sqrt{10} - 9\sqrt{10} = -7\sqrt{10}$ (Y)

15) $\sqrt{2} - 18\sqrt{7} - 5\sqrt{2} + 3\sqrt{7} = -4\sqrt{2} - 15\sqrt{7}$ (E)

4) $10\sqrt{5} + \sqrt{5} = 11\sqrt{5}$ (T)

16) $-9\sqrt{c} + 4\sqrt{d} - 4\sqrt{c} + 2\sqrt{d} = -13\sqrt{c} + 6\sqrt{d}$ (A)

5) $-3\sqrt{10} - 8\sqrt{10} = -11\sqrt{10}$ (O)

6) $7\sqrt{x} - 15\sqrt{x} = -8\sqrt{x}$ (N)

7) $\sqrt{8} + \sqrt{50} = 2\sqrt{2} + 5\sqrt{2} = 7\sqrt{2}$ (D)

17) $\sqrt{27} + \sqrt{75} + \sqrt{12} = 3\sqrt{3} + 5\sqrt{3} + 2\sqrt{3} = 10\sqrt{3}$ (H)

8) $-\sqrt{12} + \sqrt{300} = -2\sqrt{3} + 10\sqrt{3} = 8\sqrt{3}$ (T)

18) $\sqrt{63} - \sqrt{28} + \sqrt{700} = 3\sqrt{7} - 2\sqrt{7} + 10\sqrt{7} = 11\sqrt{7}$ (V)

9) $3\sqrt{20} + 8\sqrt{45} = 6\sqrt{5} + 24\sqrt{5} = 30\sqrt{5}$ (A)

10) $\sqrt{150} - 7\sqrt{24} = 5\sqrt{6} - 14\sqrt{6} = -9\sqrt{6}$ (T)

19) $5\sqrt{8} + 9\sqrt{200} + \sqrt{32} = 10\sqrt{2} + 90\sqrt{2} + 4\sqrt{2} = 104\sqrt{2}$ (A)

11) $-2\sqrt{40} - 5\sqrt{40} = -6\sqrt{10} - 10\sqrt{10} = -16\sqrt{10}$ (N)

20) $-2\sqrt{54} + 7\sqrt{150} + 3\sqrt{144} = -6\sqrt{6} + 35\sqrt{6} + 36 = 29\sqrt{6} + 36$ (V)

12) $3\sqrt{98} - 6\sqrt{18} = 21\sqrt{2} - 18\sqrt{2} = 3\sqrt{2}$ (W)

$$\textcircled{21} \quad -4\sqrt{500} + 12\sqrt{44} - \sqrt{80}$$

$$\begin{array}{ccc} \swarrow \searrow & \swarrow \searrow & \swarrow \searrow \\ \sqrt{100} \sqrt{5} & \sqrt{4} \sqrt{11} & \sqrt{10} \sqrt{5} \\ -40\sqrt{5} & + 12\sqrt{11} & - 4\sqrt{5} = \end{array}$$

$$\boxed{-44\sqrt{5} + 12\sqrt{11} \text{ (N)}}$$

$$\textcircled{22} \quad 8\sqrt{12t} + \sqrt{300t} - 2\sqrt{27t}$$

$$\begin{array}{ccc} \swarrow \searrow & \swarrow \searrow & \swarrow \searrow \\ \sqrt{4} \sqrt{3t} & \sqrt{100} \sqrt{3t} & \sqrt{9} \sqrt{3t} \\ 16\sqrt{3t} & + 10\sqrt{3t} & - 6\sqrt{3t} = \end{array}$$

$$\boxed{20\sqrt{3t} \text{ (0)}}$$

$$\textcircled{23} \quad -10\sqrt{9t} + 3\sqrt{36t} - \sqrt{50t}$$

$$\begin{array}{ccc} \swarrow \searrow & \swarrow \searrow & \swarrow \searrow \\ \sqrt{9} \sqrt{t} & \sqrt{36} \sqrt{t} & \sqrt{25} \sqrt{2t} \\ -30\sqrt{t} & + 18\sqrt{t} & - 5\sqrt{2t} = \end{array}$$

$$\boxed{-12\sqrt{t} - 5\sqrt{2t} \text{ (E)}}$$

$$\textcircled{24} \quad 5\sqrt{160t} + 12\sqrt{75t} - 4\sqrt{250t}$$

$$\begin{array}{ccc} \swarrow \searrow & \swarrow \searrow & \swarrow \searrow \\ \sqrt{16} \sqrt{10t} & \sqrt{25} \sqrt{3t} & \sqrt{25} \sqrt{10t} \\ 20\sqrt{10t} & + 60\sqrt{3t} & - 20\sqrt{10t} \end{array}$$

$$\boxed{60\sqrt{3t} \text{ (M)}}$$

ELEVATOR ... #1

$$\textcircled{1} \quad \sqrt{2}(\sqrt{8} - 5) = \sqrt{16} - 5\sqrt{2} =$$

$$\boxed{4 - 5\sqrt{2} \text{ (m.v.)}}$$

$$\textcircled{2} \quad \sqrt{3}(1 + \sqrt{27}) = \sqrt{3} + \sqrt{81} =$$

$$\boxed{\sqrt{3} + 9 \text{ (a.T)}}$$

Pg. 215

1) $(5-\sqrt{2})(5+\sqrt{2}) = 25-2 = 23$ (O)

2) $(7+\sqrt{3})(7-\sqrt{3}) = 49-3 = 46$ (E)

3) $(\sqrt{10}-6)(\sqrt{10}+6) = 10-36 = -26$ (T)

4) $(\sqrt{10}-\sqrt{6})(\sqrt{10}+\sqrt{6}) = 10-6 = 4$ (K)

5) $(\sqrt{2}+8)(\sqrt{2}+3) = 2+8\sqrt{2}+3\sqrt{2}+24$
 $= 26+11\sqrt{2}$ (A)

6) $(\sqrt{13}+1)(\sqrt{13}-5) = 13+\sqrt{13}-5\sqrt{13}-5$
 $= 8-4\sqrt{13}$ (S)

7) $(10-\sqrt{15})(3-\sqrt{15}) = 18-3\sqrt{15}-6\sqrt{15}+15$
 $= 33-9\sqrt{15}$ (M)

8) $(9+\sqrt{7})^2 = 81+18\sqrt{7}+7 = 88+18\sqrt{7}$ (U)

9) $3\sqrt{2}(\sqrt{10}+\sqrt{2}) = 3\sqrt{20}+3\sqrt{4} = 6\sqrt{5}+6$ (D)

10) $2\sqrt{5}(7\sqrt{3}-\sqrt{10}) = 14\sqrt{15}-2\sqrt{50} = 14\sqrt{15}-10\sqrt{2}$ (L)

11) $5\sqrt{3}(2\sqrt{15}+\sqrt{8}) = 10\sqrt{45}+5\sqrt{24} = 30\sqrt{5}+10\sqrt{6}$ (C)

12) $3\sqrt{10}(4\sqrt{3}-2\sqrt{15}) = 12\sqrt{30}-6\sqrt{150} = 6\sqrt{6}-18\sqrt{10}$ (F)

13) $(8+3\sqrt{5})(1+2\sqrt{5}) = 8+3\sqrt{5}+10\sqrt{5}+6\sqrt{25}$
 $= 8+19\sqrt{5}+30$
 $= 38+19\sqrt{5}$

14) $(2\sqrt{7}+4)(5\sqrt{7}-11) = 10\sqrt{49}-22\sqrt{7}+20\sqrt{7}-44$
 $= 70-2\sqrt{7}-44$
 $= 26-2\sqrt{7}$ (H)

15 →

$$\begin{aligned} (5) \quad & (3\sqrt{10} - 5\sqrt{2})(2\sqrt{10} + 6\sqrt{2}) = \\ & = 6\sqrt{100} + 18\sqrt{20} - 10\sqrt{20} - 30\sqrt{4} \\ & = \cancel{60} + 8\sqrt{20} - \cancel{60} \\ & = 8\sqrt{20} \\ & = \boxed{16\sqrt{5} \text{ (I)}} \end{aligned}$$

Box 9 (Vampire)

⑥ $3\sqrt{2} + 8\sqrt{\frac{1}{2}}$
 $3\sqrt{2} + \frac{8\sqrt{1} \cdot \sqrt{2}}{\sqrt{2} \cdot \sqrt{2}}$
 $3\sqrt{2} + \frac{8\sqrt{2}}{2}$
 $3\sqrt{2} + 4\sqrt{2}$
 $7\sqrt{2}$

⑦ $10\sqrt{\frac{1}{5}} + \sqrt{45}$
 $\frac{10\sqrt{1} \cdot \sqrt{5}}{\sqrt{5} \cdot \sqrt{5}} + \sqrt{45}$
 $\frac{10\sqrt{5}}{5} \quad \sqrt{9} \cdot \sqrt{5}$
 $2\sqrt{5} + 3\sqrt{5}$
 $5\sqrt{5}$

⑧ $\left(\frac{7}{1}\right)^2 + \frac{\sqrt{7}}{2}$
 $\frac{2\sqrt{7}}{2} + \frac{1\sqrt{7}}{2} = \frac{3\sqrt{7}}{2}$

⑨ $\sqrt{3} - \sqrt{\frac{1}{3}}$
 $\sqrt{3} - \frac{\sqrt{1} \cdot \sqrt{3}}{\sqrt{3} \cdot \sqrt{3}}$
 $3\left(\frac{\sqrt{3}}{3}\right) - \frac{\sqrt{3}}{3}$
 $\frac{3\sqrt{3}}{3} - \frac{1\sqrt{3}}{3} = \frac{2\sqrt{3}}{3}$

⑩ $\sqrt{\frac{2}{3}} + 4\sqrt{\frac{1}{6}}$
 $\frac{\sqrt{2} \cdot \sqrt{3}}{\sqrt{3} \cdot \sqrt{3}} \quad \frac{4\sqrt{1} \cdot \sqrt{6}}{\sqrt{6} \cdot \sqrt{6}}$
 $2\left(\frac{\sqrt{6}}{3}\right) + \frac{4\sqrt{6}}{6}$
 $\frac{2\sqrt{6}}{3} + \frac{4\sqrt{6}}{6}$
 $\frac{4\sqrt{6}}{6} = \frac{\sqrt{6}}{1.5}$

⑪ $\sqrt{\frac{3}{4}} \cdot \sqrt{\frac{2}{3}} = \frac{\sqrt{6}}{\sqrt{12}} = \sqrt{\frac{1}{2}} = \frac{\sqrt{2}}{2}$
 $\frac{\sqrt{1} \cdot \sqrt{2}}{\sqrt{2} \cdot \sqrt{2}} = \frac{\sqrt{2}}{2}$

⑫ $\sqrt{\frac{7}{10}} \cdot \sqrt{\frac{2}{2}} = \frac{\sqrt{14}}{\sqrt{20}} \xrightarrow{\sqrt{4}} \frac{\sqrt{14}}{2\sqrt{5}}$
 $\frac{7 \cdot \sqrt{5}}{2\sqrt{5} \cdot \sqrt{5}} = \frac{7\sqrt{5}}{2(5)} = \frac{7\sqrt{5}}{10}$

⑬ $7\sqrt{10} - 2\sqrt{90} + 4\sqrt{\frac{1}{10}}$
 $7\sqrt{10} - 2\sqrt{9} \cdot \sqrt{10} + \frac{4\sqrt{1} \cdot \sqrt{10}}{\sqrt{10} \cdot \sqrt{10}}$
 $7\sqrt{10} - 6\sqrt{10} + \frac{2\sqrt{10}}{10} = \frac{2\sqrt{10}}{5}$
 $5\left(\frac{\sqrt{10}}{5}\right) + \frac{2\sqrt{10}}{5}$
 $\frac{5\sqrt{10}}{5} + \frac{2\sqrt{10}}{5} = \frac{7\sqrt{10}}{5}$

⑭ $6\sqrt{\frac{1}{2}} - 4\sqrt{\frac{1}{8}}$
 $\frac{6\sqrt{1} \cdot \sqrt{2}}{\sqrt{2} \cdot \sqrt{2}} - \frac{4\sqrt{1} \cdot \sqrt{8}}{\sqrt{8} \cdot \sqrt{8}}$
 $\frac{6\sqrt{2}}{2} - \frac{4\sqrt{8}}{8} \xrightarrow{\sqrt{4}} \frac{3\sqrt{2}}{1} - \frac{\sqrt{2}}{1}$
 $3\sqrt{2} - \sqrt{2} = 2\sqrt{2}$

$$(15) \quad 8\sqrt{\frac{1}{4}} + \sqrt{24}$$

$$\frac{8\sqrt{1} \cdot \sqrt{6}}{\sqrt{4} \cdot \sqrt{6}}$$

$$\frac{8\sqrt{6}}{4}$$

$$4$$

$$\frac{4\sqrt{6}}{3} + \left(\frac{2\sqrt{6}}{1}\right)3$$

$$\frac{4\sqrt{6}}{3} + \frac{6\sqrt{6}}{3} = \boxed{\frac{10\sqrt{6}}{3}}$$

$$\begin{aligned} (21) \quad & -4\sqrt{500} + 12\sqrt{44} - \sqrt{80} \\ & \begin{array}{ccc} \swarrow & \searrow & \swarrow \\ \sqrt{100} & \sqrt{5} & \sqrt{4} \sqrt{11} \\ \downarrow & & \downarrow \\ -40\sqrt{5} & + 12\sqrt{11} & - 4\sqrt{5} \end{array} = \\ & \boxed{-44\sqrt{5} + 12\sqrt{11}} \quad (\text{N}) \end{aligned}$$

$$\begin{aligned} (22) \quad & 8\sqrt{12t} + \sqrt{300t} - 2\sqrt{27t} \\ & \begin{array}{ccc} \swarrow & \searrow & \swarrow \\ \sqrt{4} & \sqrt{3t} & \sqrt{100} \\ \downarrow & & \downarrow \\ 16\sqrt{3t} & + 10\sqrt{3t} & - 6\sqrt{3t} \end{array} = \\ & \boxed{20\sqrt{3t}} \quad (\text{O}) \end{aligned}$$

$$\begin{aligned} (23) \quad & -10\sqrt{4t} + 3\sqrt{36t} - \sqrt{50t} \\ & \begin{array}{ccc} \swarrow & \searrow & \swarrow \\ \sqrt{4} & \sqrt{t} & \sqrt{36} \sqrt{t} \\ \downarrow & & \downarrow \\ -30\sqrt{t} & + 18\sqrt{t} & - 5\sqrt{2t} \end{array} = \\ & \boxed{-12\sqrt{t} - 5\sqrt{2t}} \quad (\text{E}) \end{aligned}$$

$$\begin{aligned} (24) \quad & 5\sqrt{160t} + 12\sqrt{75t} - 4\sqrt{250t} \\ & \begin{array}{ccc} \swarrow & \searrow & \swarrow \\ \sqrt{16} & \sqrt{10t} & \sqrt{25} \sqrt{3t} \\ \downarrow & & \downarrow \\ 20\sqrt{10t} & + 60\sqrt{3t} & - 20\sqrt{10t} \end{array} \\ & \boxed{40\sqrt{3t}} \quad (\text{M}) \end{aligned}$$

ELEVATOR...

#10

$$\begin{aligned} (1) \quad & \sqrt{2}(\sqrt{8} - 5) = \sqrt{16} - 5\sqrt{2} = \\ & \boxed{4 - 5\sqrt{2}} \quad (\text{m}\cdot\text{V}) \end{aligned}$$

$$\begin{aligned} (2) \quad & \sqrt{3}(1 + \sqrt{27}) = \sqrt{3} + \sqrt{81} = \\ & \boxed{\sqrt{3} + 9} \quad (\text{a}\cdot\text{T}) \end{aligned}$$

$$\begin{aligned} (3) \quad & \sqrt{5}(\sqrt{15} + \sqrt{2}) = \sqrt{75} + \sqrt{10} \\ & \begin{array}{ccc} \swarrow & \searrow & \swarrow \\ \sqrt{25} & \sqrt{3} & \sqrt{2} \end{array} \\ & \boxed{5\sqrt{3} + \sqrt{10}} \quad (\text{f}\cdot\text{C}) \end{aligned}$$

$$\begin{aligned} (4) \quad & 2\sqrt{7}(\sqrt{7} - 4) = 2\sqrt{49} - 8\sqrt{7} = \\ & \boxed{14 - 8\sqrt{7}} \quad (\text{k}\cdot\text{J}) \end{aligned}$$

$$\begin{aligned} (5) \quad & 8\sqrt{3}(2\sqrt{3} + \sqrt{8}) = 16\sqrt{9} + 8\sqrt{24} = \\ & \begin{array}{ccc} \swarrow & \searrow & \swarrow \\ \sqrt{4} & \sqrt{6} & \sqrt{6} \end{array} \\ & \boxed{48 + 16\sqrt{6}} \quad (\text{n}\cdot\text{E}) \end{aligned}$$

$$\begin{aligned} (6) \quad & \sqrt{18}(\sqrt{2} - \sqrt{18}) = 5\sqrt{36} - \sqrt{324} = \\ & \boxed{30 - 18} = 12 \quad (\text{H}\cdot\text{g}) \end{aligned}$$

$$\begin{aligned} (7) \quad & (5\sqrt{2} + \sqrt{3})(\sqrt{2} + 2\sqrt{3}) = \\ & \begin{array}{l} 5(2) + \sqrt{6} + 10\sqrt{6} + 2(3) = \\ 10 + 11\sqrt{6} + 6 = \\ \boxed{16 + 11\sqrt{6}} \quad (\text{N}\cdot\text{B}) \end{array} \end{aligned}$$

$$\begin{aligned} (8) \quad & (\sqrt{5} + 9\sqrt{2})(4\sqrt{5} - \sqrt{2}) = \\ & \begin{array}{l} 4(5) + 36\sqrt{10} - \sqrt{10} - 9(2) \\ 20 + 35\sqrt{10} - 18 \\ \boxed{2 + 35\sqrt{10}} \quad (\text{k}\cdot\text{P}) \end{array} \end{aligned}$$

$$\begin{aligned} (9) \quad & (\sqrt{3} + 8)^2 = 3 + 16\sqrt{3} + 64 = \\ & \boxed{67 + 16\sqrt{3}} \quad (\text{b}\cdot\text{H}) \end{aligned}$$

$$\begin{aligned} (10) \quad & (\sqrt{6} - \sqrt{3})^2 = 6 - 2\sqrt{18} + 3 = \\ & \begin{array}{ccc} \swarrow & \searrow & \swarrow \\ \sqrt{9} & \sqrt{2} & \sqrt{2} \end{array} \\ & \boxed{9 - 6\sqrt{2}} \quad (\text{c}\cdot\text{O}) \end{aligned}$$

$$(7+\sqrt{2})(7-\sqrt{2}) = 49 - 2 = 47 \quad (18)$$

$$\underline{47 \text{ (I.F.)}}$$

$$\frac{-20}{\sqrt{16}-\sqrt{2}} \cdot \frac{\sqrt{16}+\sqrt{2}}{\sqrt{16}+\sqrt{2}} =$$

$$(12) (\sqrt{15}+\sqrt{6})(\sqrt{15}-\sqrt{6}) = 15 - 6 = 9$$

$$\underline{9 \text{ (n.T.)}}$$

$$\frac{-20\sqrt{16} - 20\sqrt{2}}{16 - 2} = \frac{-20\sqrt{16} - 20\sqrt{2}}{14} =$$

$$\underline{-5\sqrt{16} - 5\sqrt{2} \text{ (9.L.)}}$$

$$(13) \frac{4}{\sqrt{5}+\sqrt{2}} \cdot \frac{\sqrt{5}-\sqrt{2}}{\sqrt{5}-\sqrt{2}} = \frac{4\sqrt{5}-4\sqrt{2}}{5-2} =$$

$$\underline{\frac{4\sqrt{5}-4\sqrt{2}}{3} \text{ (C.T.)}}$$

$$(19) \frac{10}{\sqrt{22}+4} \cdot \frac{\sqrt{22}-4}{\sqrt{22}-4} =$$

$$\frac{10\sqrt{22}-40}{22-16} = \frac{10\sqrt{22}-40}{6}$$

$$(14) \frac{15}{\sqrt{11}-\sqrt{6}} \cdot \frac{\sqrt{11}+\sqrt{6}}{\sqrt{11}+\sqrt{6}} = \frac{15\sqrt{11}+15\sqrt{6}}{11-6} =$$

$$\underline{\frac{5\sqrt{22}-20}{3} \text{ (n.S.)}}$$

$$\frac{15\sqrt{11}+15\sqrt{6}}{5} = \frac{5(3\sqrt{11}+3\sqrt{6})}{5} =$$

$$(20) \frac{\sqrt{5}}{3-\sqrt{8}} \cdot \frac{3+\sqrt{8}}{3+\sqrt{8}} =$$

$$\underline{3\sqrt{11}+3\sqrt{6} \text{ (I.R.)}}$$

$$\frac{3\sqrt{5} + \sqrt{40}}{9-8} = \frac{3\sqrt{5} + \sqrt{40}}{1}$$

$$\downarrow \quad \downarrow$$

$$\sqrt{4} \quad \sqrt{10}$$

$$(15) \frac{9}{\sqrt{3}+1} \cdot \frac{\sqrt{3}-1}{\sqrt{3}-1} = \frac{9\sqrt{3}-9}{3-1} =$$

$$\underline{3\sqrt{5} + 2\sqrt{10} \text{ (j.N.)}}$$

$$\underline{\frac{9\sqrt{3}-9}{2} \text{ (n.D.)}}$$

$$(16) \frac{36}{4-\sqrt{7}} \cdot \frac{4+\sqrt{7}}{4+\sqrt{7}} = \frac{144+36\sqrt{7}}{16-7} =$$

$$\frac{144+36\sqrt{7}}{9} = \underline{16 + 4\sqrt{7} \text{ (n.L.)}}$$

$$(17) \frac{56}{\sqrt{3}+\sqrt{10}} \cdot \frac{\sqrt{3}-\sqrt{10}}{\sqrt{3}-\sqrt{10}} = \frac{56\sqrt{3}-56\sqrt{10}}{3-10} =$$

$$\frac{56\sqrt{3}-56\sqrt{10}}{-7} = \underline{-8\sqrt{3}+8\sqrt{10} \text{ (I.H.)}}$$