#9 (4 pgs.)

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9-5

Study Guide and Intervention

Factoring Differences of Squares

Factor $a^2 - b^2$ The binomial expression $a^2 - b^2$ is called the **difference of two squares**. The following pattern shows how to factor the difference of squares.

Difference of Squares	$a^2 - b^2 = (a - b)(a + b) = (a + b)(a - b)$.

Example 1 **Factor each binomial.**

a. $n^2 - 64$

$n^2 - 64$
$= n^2 - 8^2$ Write in the form $a^2 - b^2$.
$= (n + 8)(n - 8)$ Factor.

b. $4m^2 - 81n^2$

$4m^2 - 81n^2$
$= (2m)^2 - (9n)^2$ Write in the form $a^2 - b^2$.
$= (2m - 9n)(2m + 9n)$ Factor.

Example 2 **Factor each polynomial.**

a. $50a^2 - 72$

$50a^2 - 72$
$= 2(25a^2 - 36)$ Find the GCF.
$= 2[(5a)^2 - 6^2)]$ $25a^2 = 5a \cdot 5a$ and $36 = 6 \cdot 6$
$= 2(5a + 6)(5a - 6)$ Factor the difference of squares.

b. $4x^4 + 8x^3 - 4x^2 - 8x$

$4x^4 + 8x^3 - 4x^2 - 8x$ Original polynomial
$= 4x(x^3 + 2x^2 - x - 2)$ Find the GCF.
$= 4x[(x^3 + 2x^2) - (x + 2)]$ Group terms.
$= 4x[x^2(x + 2) - 1(x + 2)]$ Find the GCF.
$= 4x[(x^2 - 1)(x + 2)]$ Factor by grouping.
$= 4x[(x - 1)(x + 1)(x + 2)]$ Factor the difference of squares.

Exercises

Factor each polynomial if possible. If the polynomial cannot be factored, write *prime*.

1. $x^2 - 81$ — $(x + 9)(x - 9)$
2. $m^2 - 100$ — $(m + 10)(m - 10)$
3. $16n^2 - 25$ — $(4n - 5)(4n + 5)$
4. $36x^2 - 100y^2$ — $(6x + 10y)(6x - 10y)$
5. $49x^2 - 32$ — prime
6. $16a^2 - 9b^2$ — $(4a - 3b)(4a + 3b)$
7. $225c^2 - a^2$ — $(15c - a)(15c + a)$
8. $72p^2 - 50$ — $2(6p + 5)(6p - 5)$
9. $-2 + 2x^2$ — $2(x - 1)(x + 1)$
10. $-81 + a^4$ — $(a - 3)(a + 3)(a^2 + 9)$
11. $6 - 54a^2$ — $6(1 + 3a)(1 - 3a)$
12. $8y^3 - 200$ — $8(y + 5)(y - 5)$
13. $4x^3 - 100x$ — $4x(x + 5)(x - 5)$
14. $2y^4 - 32y^2$ — $2y^2(y + 4)(y - 4)$
15. $8m^3 - 128m$ — $8m(m + 4)(m - 4)$
16. $6x^2 - 25$ — prime
17. $2a^3 - 98ab^2$ — $2a(a - 7b)(a + 7b)$
18. $18y^2 - 72y^4$ — $18y^2(1 - 2y)(1 + 2y)$
19. $169x^3 - x$ — $x(13x + 1)(13x - 1)$
20. $3a^4 - 3a^2$ — $3a^2(a + 1)(a - 1)$
21. $3x^4 + 6x^3 - 3x^2 - 6x$ — $3x(x - 1)(x + 1)(x + 2)$

Lesson 9-5

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9-5

Study Guide and Intervention *(continued)*

Factoring Differences of Squares

Solve Equations by Factoring Factoring and the Zero Product Property can be used to solve equations that can be written as the product of any number of factors set equal to 0.

Example **Solve each equation. Check your solutions.**

a. $x^2 - \frac{1}{25} = 0$

$x^2 - \frac{1}{25} = 0$ Original equation
$x^2 - \left(\frac{1}{5}\right)^2 = 0$ $x^2 = x \cdot x$ and $\frac{1}{25} = \left(\frac{1}{5}\right)\left(\frac{1}{5}\right)$
$\left(x + \frac{1}{5}\right)\left(x - \frac{1}{5}\right) = 0$ Factor the difference of squares.
$x + \frac{1}{5} = 0$ or $x - \frac{1}{5} = 0$ Zero Product Property
$x = -\frac{1}{5}$ $x = \frac{1}{5}$ Solve each equation.

The solution set is $\left\{-\frac{1}{5}, \frac{1}{5}\right\}$. Since $\left(-\frac{1}{5}\right)^2 - \frac{1}{25} = 0$ and $\left(\frac{1}{5}\right)^2 - \frac{1}{25} = 0$, the solutions check.

b. $4x^3 = 9x$

$4x^3 = 9x$ Original equation
$4x^3 - 9x = 0$ Subtract $9x$ from each side.
$x(4x^2 - 9) = 0$ Find the GCF.
$x[(2x)^2 - 3^2] = 0$ $4x^2 = 2x \cdot 2x$ and $9 = 3 \cdot 3$
$x[(2x)^2 - 3^2] = x[(2x - 3)(2x + 3)]$ Factor the difference of squares.
$x = 0$ or $(2x - 3) = 0$ or $(2x + 3) = 0$ Zero Product Property
$x = 0$ $x = \frac{3}{2}$ $x = -\frac{3}{2}$ Solve each equation.

The solution set is $\left\{0, \frac{3}{2}, -\frac{3}{2}\right\}$.

Since $4(0)^3 = 9(0)$, $4\left(\frac{3}{2}\right)^3 = 9\left(\frac{3}{2}\right)$, and $4\left(-\frac{3}{2}\right)^3 = 9\left(-\frac{3}{2}\right)$, the solutions check.

Exercises

Solve each equation. Check your solutions.

1. $81x^2 = 49$ $\left\{\frac{7}{9}, -\frac{7}{9}\right\}$
2. $36n^2 = 1$ $\left\{-\frac{1}{6}, \frac{1}{6}\right\}$
3. $25d^3 - 100 = 0$ $\{2, -2\}$
4. $\frac{1}{4}x^2 = 25$ $\{10, -10\}$
5. $36 = \frac{1}{25}x^2$ $\{-30, 30\}$
6. $\frac{49}{100} - x^2 = 0$ $\left\{-\frac{7}{10}, \frac{7}{10}\right\}$
7. $9x^3 = 25x$ $\left\{0, -\frac{5}{3}, \frac{5}{3}\right\}$
8. $7a^3 = 175a$ $\{0, -5, 5\}$
9. $2m^3 = 32m$ $\{0, -4, 4\}$
10. $16y^3 = 25y$ $\left\{0, -\frac{5}{4}, \frac{5}{4}\right\}$
11. $\frac{1}{64}x^2 = 49$ $\{-56, 56\}$
12. $4a^3 - 64a = 0$ $\{0, -4, 4\}$
13. $3b^3 - 27b = 0$ $\{0, -3, 3\}$
14. $\frac{9}{25}m^2 = 121$ $\left\{-\frac{55}{3}, \frac{55}{3}\right\}$
15. $48n^2 = 147n$ $\left\{0, -\frac{7}{4}, \frac{7}{4}\right\}$

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9-5 **Skills Practice**

Factoring Differences of Squares

Factor each polynomial, if possible. If the polynomial cannot be factored, write *prime*.

1. $a^2 - 4$
   $(a + 2)(a - 2)$
2. $n^2 - 64$
   $(n + 8)(n - 8)$
3. $1 - 49c^2$
   $(1 + 7c)(1 - 7c)$
4. $-16 + p^2$
   $(p + 4)(p - 4)$
5. $k^2 + 25$
   prime
6. $36 - 100w^2$
   $(6 - 10w)(6 + 10w)$
7. $t^2 - 81u^2$
   $(t + 9u)(t - 9u)$
8. $4h^2 - 25g^2$
   $(2h + 5g)(2h - 5g)$
9. $64m^2 - 9y^2$
   $(8m - 3y)(8m + 3y)$
10. $4c^2 - 5d^2$
    prime
11. $-49r^2 + 4t^2$
    $(2t + 7r)(2t - 7r)$
12. $8x^2 - 72p^2$
    $8(x + 3p)(x - 3p)$
13. $20q^2 - 5r^2$
    $5(2q + r)(2q - r)$
14. $32a^2 - 50b^2$
    $2(4a + 5b)(4a - 5b)$

Solve each equation by factoring. Check your solutions.

15. $16x^2 - 9 = 0$ $\left\{\pm\frac{3}{4}\right\}$
16. $25p^2 - 16 = 0$ $\left\{\pm\frac{4}{5}\right\}$
17. $36q^2 - 49 = 0$ $\left\{\pm\frac{7}{6}\right\}$
18. $81 - 4b^2 = 0$ $\left\{\pm\frac{9}{2}\right\}$
19. $16d^2 = 4$ $\left\{\pm\frac{1}{2}\right\}$
20. $18a^2 = 8$ $\left\{\pm\frac{2}{3}\right\}$
21. $s^2 - \frac{9}{25} = 0$ $\left\{\pm\frac{3}{5}\right\}$
22. $k^2 - \frac{49}{64} = 0$ $\left\{\pm\frac{7}{8}\right\}$
23. $\frac{1}{25}h^2 - 16 = 0$ $\{\pm 20\}$
24. $\frac{1}{16}y^2 = 81$ $\{\pm 36\}$

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9-5 **Practice** (Average)

Factoring Differences of Squares

Factor each polynomial, if possible. If the polynomial cannot be factored, write *prime*.

1. $k^2 - 100$
   $(k + 10)(k - 10)$
2. $81 - r^2$
   $(9 + r)(9 - r)$
3. $16p^2 - 36$
   $(4p + 6)(4p - 6)$
4. $4x^2 + 25$
   prime
5. $144 - 9f^2$
   $(12 + 3f)(12 - 3f)$
6. $36g^2 - 49h^2$
   $(6g + 7h)(6g - 7h)$
7. $121m^2 - 144n^2$
   $(11m - 12n)(11m + 12n)$
8. $32 - 8y^2$
   $8(2 - y)(2 + y)$
9. $24a^2 - 54b^2$
   $6(2a - 3b)(2a + 3b)$
10. $32s^2 - 18u^2$
    $2(4s - 3u)(4s + 3u)$
11. $9d^2 - 32$
    prime
12. $36z^3 - 9z$
    $9z(2z + 1)(2z - 1)$
13. $45q^3 - 20q$
    $5q(3q + 2)(3q - 2)$
14. $100b^3 - 36b$
    $4b(5b + 3)(5b - 3)$
15. $3t^4 - 48t^2$
    $3t^2(t + 4)(t - 4)$

Solve each equation by factoring. Check your solutions.

16. $4y^2 = 81$
    $\left\{\pm\frac{9}{2}\right\}$
17. $64p^2 = 9$
    $\left\{\pm\frac{3}{8}\right\}$
18. $98b^2 - 50 = 0$
    $\left\{\pm\frac{5}{7}\right\}$
19. $32 - 162k^2 = 0$
    $\left\{\pm\frac{4}{9}\right\}$
20. $s^2 - \frac{64}{121} = 0$
    $\left\{\pm\frac{8}{11}\right\}$
21. $\frac{16}{49} - v^2 = 0$
    $\left\{\pm\frac{4}{7}\right\}$
22. $\frac{1}{36}x^2 - 25 = 0$
    $\{\pm 30\}$
23. $27h^3 = 48h$
    $\left\{\pm\frac{4}{3}, 0\right\}$
24. $75g^3 = 147g$
    $\left\{\pm\frac{7}{5}, 0\right\}$

25. **EROSION** A rock breaks loose from a cliff and plunges toward the ground 400 feet below. The distance d that the rock falls in t seconds is given by the equation $d = 16t^2$. How long does it take the rock to hit the ground? 5 s

26. **FORENSICS** Mr. Cooper contested a speeding ticket given to him after he applied his brakes and skidded to a halt to avoid hitting another car. In traffic court, he argued that the length of the skid marks on the pavement, 150 feet, proved that he was driving under the posted speed limit of 65 miles per hour. The ticket cited his speed at 70 miles per hour. Use the formula $\frac{1}{24}s^2 = d$, where s is the speed of the car and d is the length of the skid marks, to determine Mr. Cooper's speed when he applied the brakes. Was Mr. Cooper correct in claiming that he was not speeding when he applied the brakes? 60 mi/h; yes

Answers

4 (4 pgs.)

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9-6 Study Guide and Intervention

Perfect Squares and Factoring

Factor Perfect Square Trinomials

Perfect Square Trinomial	a trinomial of the form $a^2 + 2ab + b^2$ or $a^2 - 2ab + b^2$

The patterns shown below can be used to factor perfect square trinomials.

Squaring a Binomial	Factoring a Perfect Square Trinomial
$(a + 4)^2 = a^2 + 2(a)(4) + 4^2$ $= a^2 + 8a + 16$	$a^2 + 8a - 16 = a^2 - 2(a)(4) + 4^2$ $= (a + 4)^2$
$(2x - 3)^2 = (2x)^2 - 2(2x)(3) + 3^2$ $= 4x^2 - 12x + 9$	$4x^2 - 12x + 9 = (2x)^2 - 2(2x)(3) + 3^2$ $= (2x - 3)^2$

Example 1 **Determine whether $16n^2 - 24n + 9$ is a perfect square trinomial. If so, factor it.**

Since $16n^2 = (4n)(4n)$, the first term is a perfect square.

Since $9 = 3 \cdot 3$, the last term is a perfect square.

The middle term is equal to $2(4n)(3)$.

Therefore, $16n^2 - 24n + 9$ is a perfect square trinomial.

$16n^2 - 24n + 9 = (4n)^2 - 2(4n)(3) + 3^2$
$= (4n - 3)^2$

Example 2 **Factor $16x^2 - 32x + 15$.**

Since 15 is not a perfect square, use a different factoring pattern.

$16x^2 - 32x + 15$	Original trinomial
$= 16x^2 + mx + nx + 15$	Write the pattern.
$= 16x^2 - 12x - 20x + 15$	$m = -12$ and $n = -20$
$= (16x^2 - 12x) - (20x - 15)$	Group terms.
$= 4x(4x - 3) - 5(4x - 3)$	Find the GCF.
$= (4x - 5)(4x - 3)$	Factor by grouping.

Therefore $16x^2 - 32x + 15 = (4x - 5)(4x - 3)$.

Exercises

Determine whether each trinomial is a perfect square trinomial. If so, factor it.

1. $x^2 - 16x + 64$
   yes; $(x - 8)(x - 8)$
2. $m^2 + 10m + 25$
   yes; $(m + 5)(m + 5)$
3. $p^2 + 8p + 64$
   no

Factor each polynomial if possible. If the polynomial cannot be factored, write *prime*.

4. $98x^2 - 200y^2$
   $2(7x + 10y)(7x - 10y)$
5. $x^2 + 22x + 121$
   $(x + 11)^2$
6. $81 + 18s + s^2$
   $(9 + s)^2$
7. $25c^2 - 10c - 1$
   prime
8. $169 - 26r + r^2$
   $(13 - r)^2$
9. $7x^2 - 9x + 2$
   $(7x - 2)(x - 1)$
10. $16m^2 + 48m + 36$
    $4(2m + 3)^2$
11. $16 - 25a^2$
    $(4 + 5a)(4 - 5a)$
12. $b^2 - 16b + 256$
    prime
13. $36x^2 - 12x + 1$
    $(6x - 1)^2$
14. $16a^2 - 40ab + 25b^2$
    $(4a - 5b)^2$
15. $8m^3 - 64m$
    $8m(m^2 - 8)$

Lesson 9-6

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9-6 Study Guide and Intervention *(continued)*

Perfect Squares and Factoring

Solve Equations with Perfect Squares Factoring and the Zero Product Property can be used to solve equations that involve repeated factors. The repeated factor gives just one solution to the equation. You may also be able to use the **square root property** below to solve certain equations.

Square Root Property	For any number $n > 0$, if $x^2 = n$, then $x = \pm\sqrt{n}$.

Example **Solve each equation. Check your solutions.**

a. $x^2 - 6x + 9 = 0$

$x^2 - 6x + 9 = 0$	Original equation
$x^2 - 2(3x) + 3^2 = 0$	Recognize a perfect square trinomial.
$(x - 3)(x - 3) = 0$	Factor the perfect square trinomial.
$x - 3 = 0$	Set repeated factor equal to 0.
$x = 3$	Solve.

The solution set is {3}. Since $3^2 - 6(3) + 9 = 0$, the solution checks.

b. $(a - 5)^2 = 64$

$(a - 5)^2 = 64$	Original equation
$a - 5 = \pm\sqrt{64}$	Square Root Property
$a - 5 = \pm 8$	$64 = 8 \cdot 8$
$a = 5 \pm 8$	Add 5 to each side.
$a = 5 + 8$ or $a = 5 - 8$	Separate into 2 equations.
$a = 13$ $a = -3$	Solve each equation.

The solution set is {−3, 13}. Since $(-3 - 5)^2 = 64$ and $(13 - 5)^2 = 64$, the solutions check.

Exercises

Solve each equation. Check your solutions.

1. $x^2 + 4x + 4 = 0$ {−2}
2. $16n^2 + 16n + 4 = 0$ $\left\{-\frac{1}{2}\right\}$
3. $25d^2 - 10d + 1 = 0$ $\left\{\frac{1}{5}\right\}$
4. $x^2 + 10x + 25 = 0$ {−5}
5. $9x^2 - 6x + 1 = 0$ $\left\{\frac{1}{3}\right\}$
6. $x^2 + x + \frac{1}{4} = 0$ $\left\{-\frac{1}{2}\right\}$
7. $25k^2 + 20k + 4 = 0$ $\left\{-\frac{2}{5}\right\}$
8. $p^2 + 2p + 1 = 49$ {−8, 6}
9. $x^2 + 4x + 4 = 64$ {−10, 6}
10. $x^2 - 6x + 9 = 25$ {−2, 8}
11. $a^2 + 8a + 16 = 1$ {−3, −5}
12. $16y^2 + 8y + 1 = 0$ $\left\{-\frac{1}{4}\right\}$
13. $(x + 3)^2 = 49$ {−10, 4}
14. $(y + 6)^2 = 1$ {−7, −5}
15. $(m - 7)^2 = 49$ {0, 14}
16. $(2x + 1)^2 = 1$ {−1, 0}
17. $(4x + 3)^2 = 25$ $\left\{-2, \frac{1}{2}\right\}$
18. $(3h - 2)^2 = 4$ $\left\{\frac{4}{3}, 0\right\}$
19. $(x + 1)^2 = 7$ $\{-1 \pm \sqrt{7}\}$
20. $(y - 3)^2 = 6$ $\{3 \pm \sqrt{6}\}$
21. $(m - 2)^2 = 5$ $\{2 \pm \sqrt{5}\}$

Answers

#6 cont.

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9-6

Skills Practice

Perfect Squares and Factoring

Determine whether each trinomial is a perfect square trinomial. If so, factor it.

1. $c^2 - 6c + 9$
   yes; $(c - 3)^2$
2. $r^2 + 4r + 4$
   yes; $(r + 2)^2$
3. $g^2 - 14g + 49$
   yes; $(g - 7)^2$
4. $2w^2 - 4w + 9$
   no
5. $4d^2 - 4d + 1$
   yes; $(2d - 1)^2$
6. $9n^2 + 30n + 25$
   yes; $(3n + 5)^2$

Factor each polynomial, if possible. If the polynomial cannot be factored, write *prime*.

7. $2x^2 - 72$
   $2(x + 6)(x - 6)$
8. $6b^2 + 11b + 3$
   $(2b + 3)(3b + 1)$
9. $36t^2 - 24t + 4$
   $4(3t - 1)^2$
10. $4h^2 - 56$
    $4(h^2 - 14)$
11. $17a^2 - 24ac$
    $a(17a - 24c)$
12. $q^2 - 14q + 36$
    prime
13. $y^2 + 24y + 144$
    $(y + 12)^2$
14. $6d^2 - 96$
    $6(d - 4)(d + 4)$
15. $4k^2 + 12k + 9$
    $(2k + 3)^2$
16. $6x^2 + 28x - 10$
    $2(x + 5)(3x - 1)$

Solve each equation. Check your solutions.

17. $x^2 - 18x + 81 = 0$ $\{9\}$
18. $4p^2 + 4p + 1 = 0$ $\left\{-\frac{1}{2}\right\}$
19. $9g^2 - 12g + 4 = 0$ $\left\{\frac{2}{3}\right\}$
20. $y^2 - 16y + 64 = 81$ $\{-1, 17\}$
21. $4n^2 - 17 = 19$ $\{\pm 3\}$
22. $x^2 + 30x + 150 = -75$ $\{-15\}$
23. $(k + 2)^2 = 16$ $\{-6, 2\}$
24. $(m - 4)^2 = 7$ $\{4 \pm \sqrt{7}\}$

Lesson 9-6

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9-6

Practice (Average)

Perfect Squares and Factoring

Determine whether each trinomial is a perfect square trinomial. If so, factor it.

1. $m^2 + 16m + 64$
   yes; $(m + 8)^2$
2. $9s^2 - 6s + 1$
   yes; $(3s - 1)^2$
3. $4y^2 - 20y + 25$
   yes; $(2y - 5)^2$
4. $16p^2 + 24p + 9$
   yes; $(4p + 3)^2$
5. $25b^2 - 4b + 16$
   no
6. $49k^2 - 56k + 16$
   yes; $(7k - 4)^2$

Factor each polynomial, if possible. If the polynomial cannot be factored, write *prime*.

7. $3p^2 - 147$
   $3(p + 7)(p - 7)$
8. $6x^2 + 11x - 35$
   $(2x + 7)(3x - 5)$
9. $50q^2 - 60q + 18$
   $2(5q - 3)^2$
10. $6t^3 - 14t^2 - 12t$
    $2t(3t + 2)(t - 3)$
11. $6d^2 - 18$
    $6(d^2 - 3)$
12. $30k^2 + 38k + 12$
    $2(5k + 3)(3k + 2)$
13. $15b^2 - 24bc$
    $3b(5b - 8c)$
14. $12h^2 - 60h + 75$
    $3(2h - 5)^2$
15. $9n^2 - 30n - 25$
    prime
16. $7u^2 - 28m^2$
    $7(u - 2m)(u + 2m)$
17. $w^4 - 8w^2 - 9$
    $(w^2 + 1)(w + 3)(w - 3)$
18. $16c^2 + 72cd + 81d^2$
    $(4c + 9d)^2$

Solve each equation. Check your solutions.

19. $4k^2 - 28k = -49$
    $\left\{\frac{7}{2}\right\}$
20. $50b^2 + 20b + 2 = 0$
    $\left\{-\frac{1}{5}\right\}$
21. $\left(\frac{1}{2}t - 1\right)^2 = 0$
    $\{2\}$
22. $g^2 + \frac{2}{3}g + \frac{1}{9} = 0$
    $\left\{-\frac{1}{3}\right\}$
23. $p^2 - \frac{6}{5}p + \frac{9}{25} = 0$
    $\left\{\frac{3}{5}\right\}$
24. $x^2 + 12x + 36 = 25$
    $\{-11, -1\}$
25. $y^2 - 8y + 16 = 64$
    $\{-4, 12\}$
26. $(h + 9)^2 = 3$
    $\{-9 \pm \sqrt{3}\}$
27. $w^2 - 6w + 9 = 13$
    $\{3 \pm \sqrt{13}\}$

28. **GEOMETRY** The area of a circle is given by the formula $A = \pi r^2$, where r is the radius. If increasing the radius of a circle by 1 inch gives the resulting circle an area of 100π square inches, what is the radius of the original circle? 9 in.

29. **PICTURE FRAMING** Mikaela placed a frame around a print that measures 10 inches by 10 inches. The area of just the frame itself is 69 square inches. What is the width of the frame? 1.5 in.